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Examiners' Report
Principal Examiner Feedback

Summer 2022

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In Further Pure Mathematics (4PM1)

Paper 02R

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Introduction

It appeared that the majority of candidates were familiar with the content of the specification as generally all questions were attempted. Candidates generally attempted all questions on the paper, however the presentation of some work was less clearly structured than in some previous exam series.

Candidates should be encouraged to read questions carefully to ensure that they follow instructions such as 'hence' or 'use algebra to'. They should also pay attention to any requirements relating to the form for the solution.

Question 1

In part (a) of the question candidates were asked to find a position vector of B from the position vector of point A and the vector $\underset{AB}{\rightarrow}$. This question was well attempted by the majority of candidates, with most able to give a correct expression for the position vector of B in the required form. A common mistake seen was $6i + 8j - (3i - 2j) = 3i + 10j$ (subtracting rather than adding vector $\underset{AB}{\rightarrow}$). Some candidates worked out the correct vector, but did not give it in the required form.

In part (b) candidates were required to find the magnitude of vector $\underset{AB}{\rightarrow}$. This was generally answered well. A common error was to find the magnitude of the answer to part (a). Some candidates did not seem to know what the magnitude was, and wrote scalar multiples of their answers to part (a).

In part (c) of the question candidates were asked to find a unit vector that is parallel to $\underset{AB}{\rightarrow}$. If the candidate scored well in parts (a) and (b), then they generally scored well here too. Occasionally, the candidate who found the magnitude of their OB vector in part b, would try to use it again here. It was evident, however, that many candidates were uncertain what constituted a unit vector.

Question 2

This question required candidates to make use of the chain rule. Candidates who understood what needed to be done to perform well on this question, frequently scored full marks.

The vast majority of candidates were able to efficiently arrive at the correct expression for the volume as a start point to their answer. Common mistakes seen were $3x^2$ or calculating the surface area. Those who correctly stated $V = 3x^3$ almost always went on to differentiate this correctly.

The majority of candidates were able to correctly state the chain rule that was needed to answer the question, most using the notation in the mark scheme. Those candidates who did not have the correct chain rule generally had some confusion with their h which they tried to include in the chain rule, albeit incorrectly. Candidates who were able to correctly state the chain rule were generally able to go on to give a fully correct answer. The most successful candidates gave the chain rule as $\frac{dx}{dt} =$, those who did not give the chain rule in this format sometimes made errors in later substitutions or rearrangements. Some candidates did not show their substitution and in a small number of cases despite having a correct expression for $\frac{dx}{dt}$ failed to arrive at the correct final answer. Most candidates gave the answer of $\frac{2}{9}$ or rounded to 0.22. It was rare to see candidates give a less accurate answer of 0.2.

Question 3

This question required candidates to work with information about the third and fifth terms of a geometric series to find the first term and common ratio. They then needed to find the sum to infinity of the series.

This question was generally well answered, with many candidates able to give a fully correct answer to both parts (a) and (b). Once candidates had been able to use the information about the third and fifth terms to form simultaneous equations in a and r they were generally able to go on to solve these. Some errors were seen in rearrangement, and a minority of candidates did not reject a negative value of r where this had been found.

In part (b) candidates were generally able to use the values of a and r to obtain a value for the sum to infinity of the series before rationalising the denominator to give the answer in the required form. A small number of candidates either did not know how to rationalize the denominator or did not identify that they had not given the answer in the form indicated in the question. Where incorrect values of r and/or a had been found in part (a) of the question candidates often went on to correctly substitute these into the formula for the sum to infinity of the series, however in some cases candidates did not consider the requirement for $|r|<1$ when using this formula.

Question 4

In part (a) of this question candidates were expected to make use of the factor and remainder theorems to obtain two equations which could then be solved simultaneously. In part (b) candidates needed to perform polynomial division (or find the quadratic factor by some other method) prior to demonstrating that, for f(x) as given in the question, f(x)=0 has only one root.

In attempting part (a), most candidates utilised the factor theorem and remainder theorem as was anticipated. Candidates attempting these were generally able to form two correct equations from putting f(-1) = 0 and f(-2) = - 5, although some errors in arithmetic were seen. Once two equations were obtained the majority of candidates attempted to solve these simultaneously, although a minority appeared not to know how to do this. Attempts at solving the equations simultaneously were generally successful, however, it was not uncommon to see the two equations added together rather than subtracted and there were examples seen where candidates made mistakes in subtracting the equations, leading to incorrect values for p and q. A number of candidates found non-integer values for p and/or q. Since the question is clear that they are both integers this should have warned candidates that they had made an error.

A minority of candidates attempted polynomial division for part (a). Where this was attempted some were able to correctly formulate equations based on the results of these divisions (the factor and the remainder given in the question) and go on to solve these to obtain the correct values of p and q. Others were not able to identify the equations that they needed from their attempts at division or made significant errors in the attempted division due to the presence of p and q in the f(x) they were attempting to divide.

Part (b) of the question was slightly less successfully completed. Most candidates made an attempt at long division to obtain the quadratic factor. Many candidates were able to complete this algebraic division correctly. Use of the determinant to assess the number and nature of any roots was commonplace as was also simply using the quadratic formula and finding no real solutions, only a minority used completing the square to show that there were no real roots of the quadratic. A few candidates incorrectly tried to use the determinant on the cubic by ignoring the term in x^3.
A sizeable minority of those who had obtained the correct quadratic factor did not know how to show that f(x) had only one root. Some assumed that producing the quadratic factor was sufficient, others wrote their conclusion without giving any reasons to support it.

Question 5

This question required candidates to complete the table of values for $y = e^{3x-2}$, plot the graph and then identify a suitable line to draw onto their graph to solve a given equation.

In part (a) candidates were asked to complete the table of values. This was done correctly by the vast majority of candidates, with only a minority having errors such as not rounding the values to the required accuracy.

Part (b) was also generally well answered, with most candidates able to accurately plot the five point from the table of values and join these with a smooth curve. Some candidates made errors in plotting which were possibly due to misinterpreting the scale (which often led to more than one point being plotted incorrectly) or a slip which was not identified by the candidate. There were a minority of candidates who thought it appropriate to join the points with straight line segments.

Part (c) differentiated well between candidates. Some were unable to make a start through an appropriate rearrangement, where others were able to fully complete the question accurately. The majority of candidates were able to reach the 3 – x and identify that they should therefore plot y = 3 – x. Generally, the line, once identified, was plotted correctly, however there were some errors in reading the scale leading to incorrect lines being drawn. Once the correct line had been drawn candidates generally seemed aware of how to obtain the required estimated solution, however there were some errors - x given to more than 1 decimal place, stating the y-value instead of the x-value or giving a coordinate as the answer.

Question 6

This question asked candidates to find an angle within a pyramid by use of Pythagoras' theorem and trigonometry in 3D. There were a variety of different approaches that could be followed in answering the question and these were seen in the responses candidates gave.

For well-prepared candidates with the ability to think in 3D space required, and then reduce to two 2D problems, this straightforward 3D Pythagoras and trigonometry question presented no problems. Many very efficient neatly constructed methods to find the required angle were presented. Where students found this question more challenging they were often able to make some progress by correctly identifying the angle required, labelling sides appropriately and beginning to work with these values and for use of appropriate trigonometry (often with incorrect lengths). Some candidates did not make use of the diagram or make clear which angle and lengths they were working with which would probably have helped them to clarify their thinking and avoid some of the errors that were seen in their working.

Of the many candidates who were able to get started, a variety of approaches were adopted to find the correct angle, including use of the cosine and sine rules, even though much of the work was being done in right-angled triangles. A few candidates did not handle Pythagoras' theorem efficiently.
Answers that included diagrams with two separate triangles, typically one in the plane VCD, usually the right-angled triangle with the right angle at the midpoint of CD, and one in the plane perpendicular to this and the base, showed clarity of work and were usually accompanied by the working required to award the method marks even where correct answers were not achieved.

Question 7

This question required candidates to solve a variety of different trigonometric equations.

Part (a) of this question was often answered well with candidates generally able to find one solution (x=15) and many able to go on to identify all three solutions in the given range.

Surprisingly few candidates explicitly changed the range to indicate that they were considering all values of 3x-15 between -15 and 525 degrees and finding values for inverse cos accordingly. Some candidates attempted to use the addition rule for cosine and did not make meaningful progress towards finding solutions.

Part (b) was found to be more challenging – most candidates used the identity $\tan y = \frac{\sin y}{\cos y}$, however many then divided through by sin y (or by tan y depending on how they had applied the identity) and therefore lost some of the solutions of the equation in the given range. Another common error was failing to find more than a single solution to cosx = -3/4.

In part (c) candidates needed to use the identity for $\cos^2 \theta$, obtain and then solve a three term quadratic in cosine, and find the appropriate values. This part of the question was generally answered well. A good number of candidates made some progress with this part of the question, with most recognising the identity required, making the necessary substitution and going on to solve their three term quadratic in cosine which in most cases led to the candidate giving all three correct answers in the given interval. However, it was common to see incorrect attempts to solve, or candidates obtaining some but not all of the angles in range. A minority of candidates found this part of the question challenging and used an incorrect 'identity' that they had created.

Question 8

In part (a) of this question candidates were required to show that the x-coordinate of the point of intersection of the two curves satisfied a given equation and then solve this to show that the x-coordinate of this point was $\frac{1}{3}\ln 9$. Parts (a)(i) and (a)(ii) of this question were often answered well by candidates.

In part (a)(i) only a small number of candidates did not appear to know how to start, the majority of candidates were able to equate the given expressions. From here many candidates went down the route of collecting the integer terms to arrive at some form of $e^{3x} = 10 - 9e^{-3x}$ and were able to correctly deal with the e^{-3x}. A small number of candidates did not realise e^{-3x} is actually $\frac{1}{e^{3x}}$ and therefore did not make any progress on part i) of this question. There were a minority of candidates who equated the given expressions and then seemingly did not know how to rearrange these to obtain the given equation, often simply writing down the given solution without showing any working. Candidates should understand that a question that asks them to 'show that' will require clear working to support the result given.

A relatively common approach was to let $e^{-3x} = y$ and form the corresponding quadratic equation in y and then change back to form the correct equation in terms of e^{3x}. A small but notable number of candidates using this approach failed to write their quadratic equation in y in terms of e^{3x} and thus did not score the final mark in i); often this was because they progressed straight into factorising their equation in y to find the solution to part aii).

In part (a)(ii) most could solve to find e^{3x} and then the required value of x, although some did not reject the second value of x. Most candidates, however, if finding $x = 0$, highlighted that this was the origin and thus A must be $x = \frac{1}{3}\ln 9$.

In part (b) of the question candidates were asked to find the area of the finite region bounded by the two curves shown. This was less successfully attempted than part (a). Many candidates were able to correctly identify the requirement for the difference between the two expressions, however some candidates incorrectly attempted to use the quadratic found at the conclusion of (a)(i) as a substitution.

Where candidates attempted the correct integrations then there were some very good solutions. A minority of candidates had a sign error on one of their terms in the integration due to a missing bracket; $9 - 9e^{-3x} - e^{3x} - 1$. It was very common for candidates to correctly integrate the terms given, if however mistakes were made this was usually dealing with either of the exponential terms and almost always forgetting to divided by 3 or -3; $-9e^{-3x}$ and $-e^{3x}$ were incorrect terms seen in attempts at integration.

When substituting the limits some candidates failed to show the substitution of $x = 0$, incorrectly assuming that this would give 0. Many did not read the question and thus gave their answer as a decimal 1.99… rather than the exact area, others attempted to work with exact values but made errors in their working.

Question 9

Part (a) of this question asked candidates to write $\frac{3}{(3-x)^3}$ in the form $a(1 - bx)^{-3}$. In part (b) they were then asked to expand this in ascending powers of *x* up to and including the term in x^3. Finally, in part (c), candidates were asked to use a suitable value of *x* to make an approximation for $\frac{24}{125}$ correct to 5 decimal places and then calculate the percentage error.

Part (a) of this question caused challenge to a significant proportion of candidates. Candidates were more likely to be able to correctly find the value of *b* than they were *a*. The most common mistake was to take out a factor of 3 rather than 3^{-3} leaving 9 and not 1/9 for the value of a.

Many were still able to make a reasonable attempt at the binomial expansion in (b) although some did not use *their* $-\frac{x}{3}$ in substitutions. The majority were able to complete a correct attempt at binomial expansion as outlined in the mark scheme and it was common for candidates to gain the first 2 marks in this part, usually falling down only from an incorrect value for a. Candidates did occasionally go astray in calculating the algebraic terms but these were not common. Some candidates did not correctly use $(-\frac{x}{3})$ in their expansion or forgot the denominators. A minority of candidates did not use part (a) to help with part (b) and attempted to expand the original expression.

Part (c) proved challenging for many candidates, although 1 or 2 marks were awarded where candidates showed a substitution of *their x* into *their* binomial expansion or used the correct calculation for the % error. A common error was for candidates to incorrectly substitute $\frac{24}{125}$ into their expression from (b), others put their expansion equal to 24/125 and attempted to solve for x. A very significant number of candidates did not demonstrate an understanding of the fact that percentage change was change/original x100. Most of them instead replaced change with new amount. The most able candidates were able to attempt this question efficiently and with little to no mistakes.

Question 10

This question required candidates to identify features of a given curve – the equations of asymptotes (part a) and coordinates of the points of intersection of the curve with the coordinate axes (b). The candidates were then asked to use calculus to show that at every point on the curve the gradient was negative (part c). Once candidates had identified these features they were asked to sketch the curve (part d) and work with information about the normal to the curve at point A to find the coordinates of the point where this normal intersected the curve for a second time (part e).

This question differentiated well between candidates. There were a good number of attempts at the question and a full spread of marks across those available was seen.

There were a good number of correct answers to (a) and (b) – asymptotes and intercepts for the given graph. In part (a) the most frequent error was simply to state the values and not give the asymptotes as equations. In part (b) candidates often did not give the coordinates of the points of intersection but rather the x-coordinate for the y-intercept and the y-coordinate for the x-intercept, this was condoned.

In (c) candidates generally attempted differentiation, although there were varying degrees of success in terms of the accuracy of their technique. Many candidates were able to score 3 marks here, but the final mark proved more challenging as justifications were often missing or incomplete.

When attempting differentiation, the quotient rule was seen in almost all cases with the product rule only rarely seen. Generally, candidates were able to give a correct initial quotient rule, however errors appeared during the final simplification; a common error seen was incorrect simplification of the expression $7(2x - 3) - 2(7x - 2)$ to give -25 rather than the correct value of -17. A small number of candidates had the terms in the numerator the wrong way around. The product rule was seen and correctly used by a small number of candidates. An error observed in product rule was making a mistake with the negative power of $(2x - 3)$ (which should have been -2).

To complete the answer to part (c) candidates needed to justify that the gradient was negative for all values of x. Most candidates attempted an explanation here, although it appeared some believed the results of the differentiation alone were sufficient to answer this. Candidates came up with many creative explanations as to why the gradient should necessarily be negative though not all of these were sufficient to gain credit here. Most candidates identified that the numerator of the derived function was negative, but a common error was to fail to consider the denominator. Some candidates tried to verify the negativity of the derivative by substituting one or several values for x.

In part (d) there were some very good sketches of the graph seen which included all of the required labels, however this was not the most common response. Incorrect graphs, some of which had completely the wrong shape, were common. Common errors included – only one branch drawn (top right was often omitted), branches drawn in the wrong "quadrants", asymptotes not labelled, intersections with axes not labelled or labelled on the wrong axis.

Part (e) proved challenging to many candidates with 0 or 1 (for the final M mark) being common to award and some candidates made little or no attempt at this part of the question. There were, however, still a pleasing number of fully correct or almost fully correct responses which were often well set out. Candidates who had an idea how to attempt this generally equated their answer to part c with -1/17. Some progressed further by working towards a 3TQ from which the coordinates of A were correctly determined. It was also common to see candidates realise that this could be easily answered by realising that $(2x – 3)^2 = 289$ became $2x – 3 = \pm 17$ etc. Candidates without the correct coordinates would often progress no further. In subsequent working some candidates incorrectly used x = -7 instead but continued to find the line using the gradient of 17 and thus gained method marks. Candidates who then equated the line to part c) often proceeded to the correct exact answer.

Question 11

In this question candidates were required to use information about the surface area of a cylinder together with the formula for volume of a cylinder to show that $V = 300r - \pi r^3$ for this cylinder (part a). They were then asked to use calculus to show the exact value of *r* which makes *V* a maximum value and justify that this was a maximum value of *V* (part b). Finally, the candidates were told that the volume of the cylinder was the same as that of a sphere and asked to find the greatest possible radius of the sphere (part c).

This question was attempted by the majority of candidates.

There were a high proportion of correct answers seen in (a) although there were also instances of candidates working with incorrect expressions and forcing these to give the required result. Most candidates correctly stated the formula for the surface area of a cylinder; where a mistake was made it was in stating π r2 rather than 2π r2 or πrh and not 2πrh. Most candidates subsequently arrived at and successfully substituted an expression for h into the formula for the volume and most of these continued to the given result.

In (b)(i) most candidates were able to make an attempt at differentiation and many went on to show the required value for r. Where both negative and positive values for r were given, many forgot to reject the negative value, unfortunately. Part (b)(ii) was not answered quite as well, with some not identifying a suitable method and others differentiating correctly but not giving sufficient justification for the second mark. It was important finish this part with a conclusion as the question asked for a justification and some candidates did not do this in an acceptable manner.

Part (c) proved relatively challenging. A common error was to not realise that the original r and the radius being described here were not the same and to form and solve an equation from equating volume formulae with a single unknown r. Some candidates had the correct idea, but made errors in substitution or, more commonly, in solving for p.

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